MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2017/2018

DIM5068 – MATHEMATICAL TECHNIQUES 2

(For DIT students only)

14 MARCH 2018
09.00 am - 11.00 am
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of **2** pages with **4** questions. Key formulae are given in the Appendix.
2. Answer **ALL** questions.
3. Write your answers in the answer booklet provided.
4. All necessary working steps must be shown.

Question 1

a) Differentiate the following functions with respect to x by using **Chain Rule.**

i) $y = \sin(x^4 - 6x^2 - 2x)$. (5 marks)

ii) $y = -\dfrac{4}{\sqrt[3]{3x^2 + 5x}}$. (6 marks)

b) Differentiate $9y^2 - 5x^4 + 3xy^2 = x^3y^3$ by using implicit differentiation. (7 marks)

c) Find the coordinates of the critical points of the given function $f(x) = x^3 - x^2 - x$. (7 marks)

[TOTAL 25 MARKS]

Question 2

a) Evaluate the following definite integral.

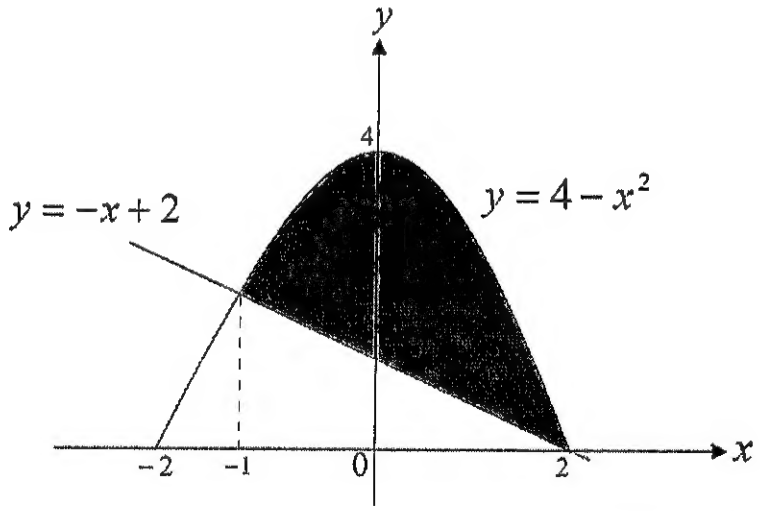
i) $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (2y^3 + \cos y)\, dy$. [Note: leave your answer in terms of π] (5 marks)

ii) $\int_0^{\pi} (3x + 5)\sin x\, dx$. [Hint: use **Integration by Parts**] (8 marks)

b) Find the indefinite integral $\int (3x^4 - 7)^6 (12x^3)\, dx$.

[Hint: use **Substitution Method**] (5 marks)

c) Find the area of the region bounded by the graphs of $y = 4 - x^2, y = -x + 2, x = -1, x = 2$ as shown below. (7 marks)



[TOTAL 25 MARKS]

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Question 3

a) Solve the first order differential equation $\frac{dy}{dx}=\frac{x^3+6x+2}{y}$, by using the **Separable Method**. (5 marks)

b) Given the differential equation $\frac{dy}{dx}+\frac{y}{x}=\frac{29}{x}$.

i) Identify the $p(x)$ and $q(x)$. (2 marks)

ii) Calculate the integrating factor, μ. (1.5 marks)

iii) Find y given $\mu y=\int \mu q(x)\,dx$. (3.5 marks)

iv) From your answer in part b(iii), determine the solution of the initial value problem if $y(7)=33$. (2 marks)

v) State the general solution of y. (1 mark)

c) Given the non-homogeneous differential equation $8y''-6y'+y=29x+4$.

i) Determine the complementary solution, y_c. (3 marks)

ii) Compute the particular solution, y_p. (7 marks)

[TOTAL 25 MARKS]

Question 4

a) Let $\mathbf{a}=2\mathbf{i}+4\mathbf{j}-2\mathbf{k}$ and $\mathbf{b}=7\mathbf{i}+4\mathbf{j}+5\mathbf{k}$.

i) Compute $3\mathbf{b}\cdot(-2\mathbf{a})$. (4 marks)

ii) Find the value of x and y if $\mathbf{a}+\mathbf{b}=\langle 9, y+3x, x\rangle$. (3 marks)

b) Given that $|\mathbf{u}|=5, |\mathbf{v}|=7$, $\mathbf{u}\cdot\mathbf{v}=3x-15$ and angle between $\mathbf{u}$ and $\mathbf{v}$ is $\frac{\pi}{2}$, find x. (3 marks)

c) Anita wants to design a card with a triangular shape. Given the vertices of the triangle $P = (2, 9, 0)$, $Q = (-4, 1, 9)$, and $R = (8, -7, 0)$.

i) Determine $\overrightarrow{PQ}$ and $\overrightarrow{PR}$. (2 marks)

ii) Calculate the cross product of $\overrightarrow{PQ}$ and $\overrightarrow{PR}$. (3 marks)

iii) Compute the total area of the card. Round your answer to 2 decimal points. (3 marks)

d) If a line passing through the points (6, 6, 2) and (7, –5, 5), compute;

i) The parametric equations of the line. (4 marks)

ii) The symmetric equations of the line. (3 marks)

[TOTAL 25 MARKS]

End of page.

APPENDIX

Derivatives: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

Differentiation Rules

General Formulae

1. $\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$
2. $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$
3. $\frac{d}{dx}(x^n) = nx^{n-1}$
4. $\frac{d}{dx}[f(u)] = \frac{dy}{du}.\frac{du}{dx}$

Exponential and Logarithmic Functions

1. $\frac{d}{dx}(e^x) = e^x$
2. $\frac{d}{dx}(a^x) = a^x \ln a$
3. $\frac{d}{dx}(\ln x) = \frac{1}{x}$
4. $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$

Trigonometric Functions

1. $\frac{d}{dx}(\sin x) = \cos x$
2. $\frac{d}{dx}(\cos x) = -\sin x$
3. $\frac{d}{dx}(\tan x) = \sec^2 x$
4. $\frac{d}{dx}(\csc x) = -\csc x \cot x$
5. $\frac{d}{dx}(\sec x) = \sec x \tan x$
6. $\frac{d}{dx}(\cot x) = -\csc^2 x$

Table of Integrals

1. $\int u\,dv = uv - \int v\,du$
2. $\int u^n\,du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$
3. $\int \frac{du}{u} = \ln|u| + C$
4. $\int e^u du = e^u + C$
5. $\int \sin u\,du = -\cos u + C$
6. $\int \cos u\,du = \sin u + C$
7. $\int \sec^2 u\,du = \tan u + C$
8. $\int \csc^2 u\,du = -\cot u + C$
9. $\int \sec u \tan u\,du = \sec u + C$
10. $\int \csc u \cot u\,du = -\csc u + C$

Application of Integrals:

Areas between Curve, $A = \int_a^b [f(x) - g(x)]\,dx$

Differential Equations

Linear Differential Equations

$$\frac{dy}{dx} + p(x)y = q(x) \quad \Rightarrow \quad \mu y = \int \mu q(x)\,dx, \text{ where } \mu = e^{\int p(x)\,dx}$$

Constant Coefficient of Homogeneous Equations

Roots of Auxiliary Equation, $r = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

General Solutions to the Auxiliary Equation:

2 Real & Unequal Roots ($b^2 - 4ac > 0$)	$y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$
Repeated Roots ($b^2 - 4ac = 0$)	$y = c_1 e^{rx} + c_2 x e^{rx}$
2 Complex Roots ($b^2 - 4ac < 0$)	$y = e^{ax}(c_1 \cos bx + c_2 \sin bx)$

Constant Coefficient of Non-Homogeneous Equations

$y = y_c + y_p$ [y_c: complementary solution, y_p: particular solution]

Vector

Length of Vector

The length of the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ is $|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

Dot Product

If θ is the angle between the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then

$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 = |\mathbf{a}||\mathbf{b}| \cos\theta$

Cross Product

If θ is the angle between the vector $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$, then

$\mathbf{a} \times \mathbf{b} = \langle a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1 \rangle$

$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin\theta$

Area for parallelogram *PQRS*

$$= \left|\vec{PQ} \times \vec{PR}\right|$$

Area for triangle PQR

$$= \frac{1}{2}\left|\vec{PQ} \times \vec{PR}\right|$$

Equation of Lines

Vector equation: $\mathbf{r} = \mathbf{r}_0 + \mathbf{tv}$

Parametric equations: $x = x_0 + at \quad y = y_0 + bt \quad z = z_0 + ct$

Symmetric equation: $\dfrac{x - x_0}{a} = \dfrac{y - y_0}{b} = \dfrac{z - z_0}{c}$

Equation of Planes

Vector equation: $\mathbf{n} \cdot \mathbf{r} = \mathbf{n} \cdot \mathbf{r}_0$

Scalar equations: $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

Linear equation: $ax + by + cz + d = 0$

Angle between Two Planes: $\theta = \cos^{-1}\left(\dfrac{\mathbf{n}_1 \bullet \mathbf{n}_2}{|\mathbf{n}_1||\mathbf{n}_2|}\right)$